

Non-Extreme Black Holes from Non-Extreme Intersecting M-branes

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Abstract

We present non-extreme generalisations of intersecting p-brane solutions of eleven-dimensional supergravity which upon toroidal compactification reduce to non-extreme static black holes in dimensions $D = 4$, $D = 5$ and $6 \leq D \leq 9$, parameterized by four, three and two charges, respectively. The $D = 4$ black holes are obtained either from a non-extreme configuration of three intersecting five-branes with a boost along the common string or from non-extreme intersecting system of two two-branes and two five-branes. The $D = 5$ black holes arise from three intersecting two-branes or from a system of intersecting two-brane and five-brane with a boost along the common string. Five-brane and two-brane with a boost along one direction reduce to black holes in $D = 6$ and $D = 9$, respectively, while $D = 7$ black hole can be interpreted in terms of non-extreme configuration of two intersecting two-branes. We discuss the expressions for the corresponding masses and entropies.

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I. INTRODUCTION

Recently, black holes in string theory have become a subject of intensive research, due, in part, to the fact that microscopic properties, e.g., the statistical origin of the entropy of certain black holes can be addressed either by using a conformal field theory description of the NS-NS backgrounds [1–7] or a D -brane representation of U-dual R-R backgrounds [8–15].

By now, the classical solutions for the BPS-saturated as well as non-extreme static and rotating black holes of $N = 4, 8$ supersymmetric superstring vacua are well understood. In particular, the explicit form of the generating solution for general rotating black holes has been obtained. The generating solution is specified by the canonical choice of the asymptotic values of the scalar fields, the ADM mass, $[\frac{D-1}{2}]$ components of angular momentum and by five, three and two charges, in dimensions $D = 4$, $D = 5$ and $6 \leq D \leq 9$, respectively.¹

The explicit form of the generating solutions was first determined in the case of toroidally compactified heterotic string ($N = 4$ superstring vacua) or in the NS-NS sector of the toroidally compactified type IIA superstring.² By applying U -duality transformations such solutions are mapped onto backgrounds with R-R charges which have an interpretation in terms of D -brane configurations. These BPS-saturated black holes have regular horizons and finite semiclassical Bekenstein-Hawking (BH) entropy in dimensions $D = 4$ [23,21,24,3] and $D = 5$ [8,4]. In $D \geq 6$ the BPS-saturated axi-symmetric solutions have singular horizons and zero BH entropy [25–27,6].

Entropy of certain non-extreme or near-extreme black holes was discussed (both in the static and rotating cases) in $D = 5$ [10,28,29] and $D = 4$ [31] and for rotating black holes with NS-NS (electric) charges in $6 \leq D \leq 9$ [6].

A unifying treatment of string-theory black hole properties may arise by identifying such black holes as (toroidally) compactified configurations of intersecting two-branes and five-branes of eleven-dimensional M-theory [32,33]. The $D = 10$ backgrounds with NS-NS and R-R charges appear on an equal footing when viewed from eleven dimensions. A discussion

¹This program has been completed in $D = 5$ [16] and $D \geq 6$ [6,17]; however, in $D = 4$ only the five-charge static generating solution [18] (see also [19]) and the four-charge rotating solutions [20] were obtained. The BPS-saturated generating solutions were obtained earlier: in $D = 4$ in [21,3] and in $D = 5$ in [4]. This work was preceded by a number of papers, where special examples of such solutions were obtained (for a recent review, see [15]).

²The most general black hole configuration is obtained by applying to the generating solution a subset of T - and S -duality transformations (for $N = 4$ superstring vacua) [1] or U -duality transformations (for $N = 8$ superstring vacua) [22], which do not affect the canonical choice of the asymptotic values of the scalar fields. These transformations do not change the D -dimensional Einstein-frame metric, and thus the metric of a general black hole in this class is thus fully specified by the parameters of the generating solution. A solution with arbitrary asymptotic values of the scalar fields is found by appropriate rescalings of the physical parameters, i.e. of the ADM mass, the angular momenta and the charges, by the asymptotic values of the scalar fields.

of intersections of certain BPS-saturated M-branes along with a proposal for intersection rules was first given in [34]. A generalization to a number of different harmonic functions specifying intersecting BPS-saturated M-branes which led to a better understanding of these solutions and a construction of new intersecting p-brane solutions in $D \leq 11$ was presented in [35] (see also related work [33,36–40]). Specific configurations of that type reduce to the BPS-saturated black holes with regular horizons in $D = 5$ [35] and $D = 4$ [33] whose properties are determined by three and four charges (or harmonic functions), respectively.

The purpose of the present paper is to relate the *non-extreme* static black holes to *non-extreme* versions of intersecting M-brane solutions of [35,33]. This approach may shed light on the structure of non-extreme black holes from the point of view of M-theory, and, in particular, clarify the origin of their BH entropy. Our interpretation of non-extreme black holes as non-extreme intersecting M-branes (or p-branes in $D = 10$) does not seem to be related to the “brane-antibrane” picture suggested in [9,28,31,41].

As we shall discuss in Section II, there exists a procedure allowing one to construct a non-extreme version of a given BPS-saturated intersecting M-brane solution which generalises the approach of [35]. The resulting eleven-dimensional metric and the four-form field strength depend on the “non-extremality” parameter, representing a deviation from the BPS-saturated limit, and the “boosts”, specifying charges of the configuration. Upon dimensional reduction these parameters determine the ADM mass and the charges of non-extreme static black holes in $4 \leq D \leq 9$.³

In Sections III, IV, and V we shall consider examples of non-extreme configurations of intersecting M-branes and relate them, via dimensional reduction along internal M-brane directions, to non-extreme black holes in dimensions $D = 4$, $D = 5$ and $6 \leq D \leq 9$, respectively. In Section VI we shall present the general expressions for the mass and BH entropy of these solutions and comment on some of their consequences.

II. NON-EXTREME INTERSECTING M-BRANE SOLUTIONS

The aim is to generalise the extreme supersymmetric (BPS-saturated) intersecting M-brane solutions of [35,33] to the non-extreme case. Even though non-extreme solutions are no longer supersymmetric, it turns out that they can be constructed as a “deformation” of extreme solutions, parameterised by several one-center harmonic functions H_i , one for each constituent M-brane, and the Schwarzschild solution, parameterised by the function $f(r) = 1 - \mu/r^{D-3}$.⁴ Here D is the dimension of the space-time transverse to the configuration, and

³The non-extremality parameter μ is proportional to the ADM mass of the neutral (Schwarzschild) black hole and the “boosts” can be interpreted as parameters of symmetry transformations of the effective lower-dimensional action which generate the charged solutions when applied to the neutral black hole solution.

⁴Similar “product structure” was found previously for non-extreme versions of isotropic extreme black p-brane solutions in [42,27].

the “non-extremality” parameter μ specifies a deviation from the BPS-saturated limit. The same type of construction applies also to intersecting p-brane solutions in ten dimensions.

It should be stressed that the solutions presented in this paper should not be interpreted as intersections of non-extreme M-branes, even though they reduce to single non-extreme M-brane solutions when all other charge parameters are set equal to zero. Each of non-extreme M-branes is parameterised, in general, by independent masses and charges while the non-extreme version of intersecting M-brane solutions has only one common mass parameter – the non-extremality parameter μ . Such configurations should be viewed as non-extreme “bound-state” configurations. Note also that the non-extreme solutions below are one-parameter “deformations” of a special type of supersymmetric solutions [35,33], for which all of the harmonic functions are chosen to have a simple spherically symmetric one-center form. The non-extreme version of multi-center solutions (corresponding to individual p-branes ‘placed’ at different points in transverse space) are expected to be unstable, i.e. described by time-dependent background fields. On the contrary, the non-extreme versions of single-center solutions discussed below are static.

One way of understanding why the structure of the non-extreme solutions is similar to that of extreme ones is based on first doing a dimensional reduction to $D = 10$, applying T -duality transformation and then lifting the solution back to $D = 11$. Since a non-extreme solution has the same number of isometries as the extreme one, it can be “generated” by starting from the Schwarzschild background instead of the flat space one by T -duality considerations similar to the ones used in the extreme case in [37,38]. Alternatively, one may start with extreme solution and consider its deformation caused by turning on the non-extremality parameter μ .

A simple algorithm which leads to non-extreme version of a given extreme solution (which indeed can be checked to satisfy the eleven-dimensional supergravity equations of motion and also corresponds upon dimensional reduction to known non-extreme black hole solutions) consists of the following steps:

(1) Make the following replacements in the D -dimensional *transverse* space-time part of the metric:

$$dt^2 \rightarrow f(r)dt^2, \quad dx_n dx_n \rightarrow f^{-1}(r)dr^2 + r^2 d\Omega_{D-2}^2, \quad f(r) = 1 - \frac{\mu}{r^{D-3}}, \quad (1)$$

and also use the special one-center form of the harmonic functions,

$$H_i = 1 + \frac{Q_i}{r^{D-3}}, \quad Q_i = \mu \sinh^2 \delta_i, \quad (2)$$

for the constituent two-branes, and

$$H_i = 1 + \frac{\mathcal{P}_i}{r^{D-3}}, \quad \mathcal{P}_i = \mu \sinh^2 \gamma_i, \quad (3)$$

for the constituent five-branes.

(2) In the expression for the field strength \mathcal{F}_4 of the three-form field make the following replacements:

$$H'_i \rightarrow H'_i = 1 + \frac{Q_i}{r^{D-3} + Q_i - Q_i} = [1 - \frac{Q_i}{r^{D-3}} H_i^{-1}]^{-1}, \quad Q_i = \mu \sinh \delta_i \cosh \delta_i, \quad (4)$$

in the “electric” (two-brane) part, and

$$H_i \rightarrow H'_i = 1 + \frac{P_i}{r^{D-3}}, \quad P_i = \mu \sinh \gamma_i \cosh \gamma_i, \quad (5)$$

in the “magnetic” (five-brane) part. Here Q_i and P_i are the respective “electric” and “magnetic” charges of the configuration. In the extreme limit $\mu \rightarrow 0$, $\delta_i \rightarrow \infty$, and $\gamma_i \rightarrow \infty$, while the charges Q_i and P_i are kept fixed. In this case $\mathcal{Q}_i = Q_i$ and $\mathcal{P}_i = P_i$, so that $H'_i = H_i$. The form of \mathcal{F}_4 and the actual value of its “magnetic” part does not change compared to the extreme limit.

(3) In the case when the extreme solution has a null isometry, i.e. intersecting branes have a common string along some direction y , one can add momentum along y by applying the coordinate transformation

$$t' = \cosh \beta t - \sinh \beta y, \quad y' = -\sinh \beta t + \cosh \beta y, \quad (6)$$

to the non-extreme background obtained according to the above two steps. Then

$$\begin{aligned} -f(r)dt^2 + dy^2 &\rightarrow -f(r)dt'^2 + dy'^2 = -dt^2 + dy^2 + \frac{\mu}{r^{D-3}}(\cosh \beta dt - \sinh \beta dy)^2 \\ &= -K^{-1}(r)f(r)dt^2 + K(r)\widehat{dy}^2, \quad \widehat{dy} \equiv dy + [K'^{-1}(r) - 1]dt, \end{aligned} \quad (7)$$

$$K = 1 + \frac{\tilde{\mathcal{Q}}}{r^{D-3}}, \quad K'^{-1} = 1 - \frac{\tilde{\mathcal{Q}}}{r^{D-3}}K^{-1}, \quad \tilde{\mathcal{Q}} = \mu \sinh^2 \beta, \quad \tilde{Q} = \mu \sinh \beta \cosh \beta, \quad (8)$$

where the boost β is related to the new electric charge parameter $\tilde{\mathcal{Q}}$, i.e. momentum along direction y . In the extreme limit $\mu \rightarrow 0$, $\beta \rightarrow \infty$, the charge Q is held fixed, $K = K'$ and thus this part of the metric (7) becomes $dudv + (K - 1)du^2$, where $v, u = y \pm t$.

Below we shall illustrate this algorithm on several examples. Let us start with basic non-extreme M-brane solutions found in [43]. The two-brane background has the form⁵

$$ds_{11}^2 = T^{-1/3}(r) \left(T(r)[-f(r)dt^2 + dy_1^2 + dy_2^2] + f^{-1}(r)dr^2 + r^2 d\Omega_7^2 \right), \quad (9)$$

$$\mathcal{F}_4 = -3dt \wedge dT' \wedge dy_1 \wedge dy_2, \quad (10)$$

where

$$\begin{aligned} f &= 1 - \frac{\mu}{r^6}, \quad T^{-1} = H = 1 + \frac{\mathcal{Q}}{r^6}, \quad T' = H'^{-1} = 1 - \frac{Q}{r^6}T, \\ \mathcal{Q} &= \mu \sinh^2 \delta, \quad Q = \mu \sinh \delta \cosh \delta. \end{aligned} \quad (11)$$

⁵We shall follow [35] and use the notation T and F for the inverse powers of the harmonic functions corresponding to the two-brane and five-brane, respectively.

Again, the extreme solution is obtained by setting $f = 1$, $\mathcal{Q} = Q$, $T = T'$.⁶

The five-brane solution is

$$ds_{11}^2 = F^{-2/3}(r) \left(F(r) [-f(r) dt^2 + dy_1^2 + \dots + dy_5^2] + f^{-1}(r) dr^2 + r^2 d\Omega_4^2 \right), \quad (12)$$

$$\mathcal{F}_4 = 3 * dF'^{-1}, \quad (13)$$

where the dual form is defined with respect to the flat transverse space. The parameters of the five-brane solution are “magnetic” analogues of the “electric” parameters δ , \mathcal{Q} , Q of the two-brane solution (11) and are denoted by γ , \mathcal{P} , P , i.e.

$$f = 1 - \frac{\mu}{r^3}, \quad F^{-1} = H = 1 + \frac{\mathcal{P}}{r^3}, \quad F'^{-1} = H' = 1 + \frac{P}{r^3}, \quad (14)$$

$$\mathcal{P} = \mu \sinh^2 \gamma, \quad P = \mu \sinh \gamma \cosh \gamma.$$

The two other non-extreme solutions found in [43] correspond to two and three intersecting two-branes with equal values of parameters $\delta_i = \delta$.

A generalisation to the case of different parameters δ_i can be easily found using the above algorithm. For example, the non-extreme version of $2 \perp 2$ configuration, i.e. two two-branes intersecting at a point, is thus given by⁷

$$ds_{11}^2 = (T_1 T_2)^{-1/3} \left[-T_1 T_2 f dt^2 + T_1 (dy_1^2 + dy_2^2) + T_2 (dy_3^2 + dy_4^2) + f^{-1} dr^2 + r^2 d\Omega_5^2 \right], \quad (15)$$

$$\mathcal{F}_4 = -3dt \wedge (dT'_1 \wedge dy_1 \wedge dy_2 + dT'_2 \wedge dy_3 \wedge dy_4), \quad (16)$$

where ($i = 1, 2$)

$$f = 1 - \frac{\mu}{r^4}, \quad T_i^{-1} = 1 + \frac{\mathcal{Q}_i}{r^4}, \quad T'_i = 1 - \frac{Q_i}{r^4} T_i, \quad (17)$$

$$\mathcal{Q}_i = \mu \sinh^2 \delta_i, \quad Q_i = \mu \sinh \delta_i \cosh \delta_i.$$

For $T_1 = T_2$, $T'_1 = T'_2$ this reduces to the anisotropic four-brane solution of [43]. The non-extreme version of extreme $2 \perp 2 \perp 2$ configuration (three two-branes intersecting at a point) [34,35] has a similar form and will be discussed below in Section IV.

⁶ The relation of our notation to the notation used in [43] is the following. The radial coordinate of the D -dimensional transverse space-time in [43] is $\hat{r}^{D-3} = r^{D-3} + \mathcal{Q} = r^{D-3} H(r)$, $\mathcal{Q} \equiv r_-^{D-3}$ and $\Delta_-(\hat{r}) = H^{-1}(r)$, $\Delta_+(\hat{r}) = H^{-1}(r) f(r)$, i.e. $\Delta_{\pm}(\hat{r}) = 1 - r_{\pm}^{D-3} / \hat{r}^{D-3}$. Here again $r_+^{D-3} = \mu \cosh^2 \delta$, $r_-^{D-3} = \mu \sinh^2 \delta$, with $r_{\pm}^{D-3} \rightarrow Q$ in the extreme limit. Note also that in terms of \hat{r} , T' has the form $T' = H'^{-1} = 1 - Q / \hat{r}^{D-3}$.

⁷For the sake of simplicity, in what follows we often do not indicate explicitly the argument r (the radial transverse coordinate) of the functions T_i, F_i and f .

In the following Sections we shall construct the non-extreme configurations of intersecting M-branes which in $D \leq 9$ reproduce the generating solutions for non-extreme static black hole backgrounds. They will be built in terms of basic M-branes according to the above algorithm. As in [34,35] we shall consider only intersections which in the extreme limit preserve supersymmetry: two two-branes can intersect at a point, two five-branes can intersect at a three-brane (with three-branes allowed to intersect over a string) and five-brane and two-brane can intersect at a string.

III. $D = 4$ NON-EXTREME BLACK HOLES

Four-dimensional black holes with four independent charges⁸ can be obtained upon toroidal compactification from two different intersecting M-brane configurations [33], $2\perp 2\perp 5\perp 5$ and “boosted” $5\perp 5\perp 5$. While the two resulting black hole backgrounds are related by four-dimensional U -duality, the underlying intersecting M-brane solutions should be related by a symmetry transformation of the M-theory. Such a symmetry transformation should be obtained as a combination of T -duality and $SL(2, Z)$ symmetry of the $D = 10$ type IIB theory “lifted” to $D = 11$.

A. Intersection of two Two-Branes and two Five-Branes

The first of the above eleven-dimensional configurations corresponds to the two two-branes intersecting at a point and two five-branes intersecting at a three-brane, with each of the two-branes intersecting with each of the five-branes at a string. The non-extreme version of the BPS-saturated solution found in [33] is given by

$$ds_{11}^2 = (T_1 T_2)^{-1/3} (F_1 F_2)^{-2/3} \left[-T_1 T_2 F_1 F_2 f dt^2 + F_1 (T_1 dy_1^2 + T_2 dy_3^2) + F_2 (T_1 dy_2^2 + T_2 dy_4^2) \right. \\ \left. + F_1 F_2 (dy_5^2 + dy_6^2 + dy_7^2) + f^{-1} dr^2 + r^2 d\Omega_2^2 \right], \quad (18)$$

$$\mathcal{F}_4 = -3dt \wedge (dT'_1 \wedge dy_1 \wedge dy_2 + dT'_2 \wedge dy_3 \wedge dy_4) \\ + 3(*dF_1'^{-1} \wedge dy_2 \wedge dy_4 + *dF_2'^{-1} \wedge dy_1 \wedge dy_3). \quad (19)$$

The coordinates y_1, \dots, y_7 describe the toroidally compactified directions. The function f , parameterising a deviation from the extremality, and functions T_i, T'_i and F_i, F'_i , specifying the (non-extreme) two-brane and five-brane configurations depend on the radial coordinate r of $(1+3)$ -dimensional (transverse) space-time,

$$f = 1 - \frac{\mu}{r}, \quad T_i^{-1} = 1 + \frac{Q_i}{r}, \quad T'_i = 1 - \frac{Q_i}{r} T_i, \quad (20)$$

⁸Note that the generating solution for the most general four-dimensional static black holes of $N = 4, 8$ superstring vacua is specified by *five* independent charges.

$$\mathcal{Q}_i = \mu \sinh^2 \delta_i , \quad Q_i = \mu \sinh \delta_i \cosh \delta_i , \quad i = 1, 2 ,$$

$$F_i^{-1} = 1 + \frac{\mathcal{P}_i}{r} , \quad F_i'^{-1} = 1 + \frac{P_i}{r} , \quad (21)$$

$$\mathcal{P}_i = \mu \sinh^2 \gamma_i , \quad P_i = \mu \sinh \gamma_i \cosh \gamma_i , \quad i = 1, 2 .$$

In the extreme limit $\mu \rightarrow 0$, $\delta_i \rightarrow \infty$ and $\gamma_i \rightarrow \infty$, while the charges Q_i and P_i are held fixed. Again, in this limit $f = 1$, $T_i = T_i'$, $F_i = F_i'$.

The nine-area of the regular outer horizon $r = \mu$ of the anisotropic seven-brane metric (18) is

$$A_9 = 4\pi L^7 [r^2 (T_1 T_2 F_1 F_2)^{-1/2}]_{r=\mu} = 4\pi L^7 \mu^2 \cosh \delta_1 \cosh \delta_2 \cosh \gamma_1 \cosh \gamma_2 , \quad (22)$$

where the internal directions y_1, \dots, y_7 are assumed to have periods L . In the BPS-saturated limit the area reduces to

$$(A_9)_{BPS} = 4\pi L^7 \sqrt{Q_1 Q_2 P_1 P_2} . \quad (23)$$

Upon toroidal compactification to four dimensions one finds the following Einstein-frame metric

$$ds_4^2 = -\lambda(r) f(r) dt^2 + \lambda^{-1}(r) [f^{-1}(r) dr^2 + r^2 d\Omega_2^2] , \quad (24)$$

where

$$\lambda(r) = (T_1 T_2 F_1 F_2)^{1/2} = \frac{r^2}{[(r + \mathcal{Q}_1)(r + \mathcal{Q}_2)(r + \mathcal{P}_1)(r + \mathcal{P}_2)]^{1/2}} . \quad (25)$$

In the BPS limit $f = 1$ and $\mathcal{Q}_i \rightarrow Q_i$, $\mathcal{P}_i \rightarrow P_i$.

The four-dimensional metric (24) is precisely the one of the non-extreme four-dimensional black hole with two electric and two magnetic charges found in [44].

B. Intersection of three Five-Branes with a Boost

The second relevant configuration [33] is that of three five-branes, each pair intersecting at a three-brane, with an extra boost along a string common to three three-branes. The corresponding non-extreme background has the form

$$\begin{aligned} ds_{11}^2 = & (F_1 F_2 F_3)^{-2/3} \left[F_1 F_2 F_3 (-K^{-1} f dt^2 + K \widehat{dy}_1^2) + F_2 F_3 (dy_2^2 + dy_3^2) \right. \\ & \left. + F_1 F_3 (dy_4^2 + dy_5^2) + F_1 F_2 (dy_6^2 + dy_7^2) + f^{-1} dr^2 + r^2 d\Omega_2^2 \right] , \end{aligned} \quad (26)$$

$$\mathcal{F}_4 = 3(*dF_1'^{-1} \wedge dy_2 \wedge dy_3 + *dF_2'^{-1} \wedge dy_4 \wedge dy_5 + *dF_3'^{-1} \wedge dy_6 \wedge dy_7) , \quad (27)$$

where (cf. (7))

$$\widehat{dy}_1 = dy_1 + (K'^{-1} - 1)dt , \quad (28)$$

and K and K' depend on the boost parameter β along the string (y_1) direction. The background is thus parameterised by $f(r)$ and the following functions of r ($i = 1, 2, 3$)

$$K = 1 + \frac{\tilde{Q}}{r} , \quad K'^{-1} = 1 - \frac{\tilde{Q}}{r}K , \quad F_i^{-1} = 1 + \frac{\mathcal{P}_i}{r} , \quad F_i'^{-1} = 1 + \frac{P_i}{r} , \quad (29)$$

$$\tilde{Q} = \mu \sinh^2 \beta , \quad \tilde{Q} = \mu \sinh \beta \cosh \beta , \quad \mathcal{P}_i = \mu \sinh^2 \gamma_i , \quad P_i = \mu \sinh \gamma_i \cosh \gamma_i . \quad (30)$$

The four charges \tilde{Q} and P_1, P_2, P_3 are held fixed in the limit $\mu \rightarrow 0, \beta \rightarrow \infty, \gamma_i \rightarrow \infty$.

The area of nine-surface at $r = \mu$ is

$$A_9 = 4\pi L^7 [r^2 (K^{-1} F_1 F_2 F_3)^{-1/2}]_{r=\mu} = 4\pi L^7 \mu^2 \cosh \beta \cosh \gamma_1 \cosh \gamma_2 \cosh \gamma_3 , \quad (31)$$

$$(A_9)_{BPS} = 4\pi L^7 \sqrt{\tilde{Q} P_1 P_2 P_3} . \quad (32)$$

The four-dimensional Einstein-frame metric resulting upon toroidal compactification is of the form (24) with

$$\lambda(r) = (K^{-1} F_1 F_2 F_3)^{1/2} = \frac{r^2}{[(r + \tilde{Q})(r + \mathcal{P}_1)(r + \mathcal{P}_2)(r + \mathcal{P}_3)]^{1/2}} . \quad (33)$$

Again, this four-dimensional metric is the same as in [44], but now it depends on one electric and three magnetic charges.

Note that the dimensional reduction to ten dimensions (along y_1 direction) gives a non-extreme generalisation of a configuration of intersecting R-R p-branes of type IIA theory, namely, a zero-brane and three four-branes [36,33,39]. Applying T -duality and $SL(2, Z)$ symmetry of type IIB theory one is able to construct various other non-extreme $D = 10$ p-brane configurations which in the extreme limit have a representation in terms of intersecting D -branes. Their form is always consistent with the algorithm of Section II. In particular, it is straightforward to write down the non-extreme version of the maximally symmetric $3 \perp 3 \perp 3 \perp 3$ solution (four three-branes intersecting at a point, with each pair of three-branes intersecting at a string), found in [33,39].

IV. $D = 5$ NON-EXTREME BLACK HOLES

The extreme $D = 5$ black holes with three independent charges [8,4] (generating solution for general extreme $D = 5$ black holes with regular horizons) can be obtained from the two different intersecting M-brane configurations [35]: $2 \perp 2 \perp 2$, i.e. three two-branes intersecting at a point, and “boosted” $2 \perp 5$, i.e. intersecting two-brane and five-brane with a momentum along the common string. Below we shall present the non-extreme versions of these $D = 11$ solutions, which serve as generating solutions for non-extreme static $D = 5$ black hole solutions.

A. Intersection of three Two-Branes

This $O(4)$ symmetric background is a straightforward generalisation of the non-extreme $2\perp 2$ solution (15),(16)

$$ds_{11}^2 = (T_1 T_2 T_3)^{-1/3} \left[-T_1 T_2 T_3 f dt^2 + T_1 (dy_1^2 + dy_2^2) + T_2 (dy_3^2 + dy_4^2) + T_3 (dy_5^2 + dy_6^2) \right. \\ \left. + f^{-1} dr^2 + r^2 d\Omega_3^2 \right], \quad (34)$$

$$\mathcal{F}_4 = -3dt \wedge (dT'_1 \wedge dy_1 \wedge dy_2 + dT'_2 \wedge dy_3 \wedge dy_4 + dT'_3 \wedge dy_5 \wedge dy_6), \quad (35)$$

where

$$f = 1 - \frac{\mu}{r^2}, \quad T_i^{-1} = 1 + \frac{\mathcal{Q}_i}{r^2}, \quad T'_i = 1 - \frac{\mathcal{Q}_i}{r^2} T_i, \\ \mathcal{Q}_i = \mu \sinh^2 \delta_i, \quad Q_i = \mu \sinh \delta_i \cosh \delta_i, \quad i = 1, 2, 3. \quad (36)$$

For $\delta_1 = \delta_2 = \delta_3$, i.e. equal T_i and equal T'_i , this solution coincides with the anisotropic six-brane solution of [43].

The nine-area of the regular horizon at $r = \mu^{1/2}$ is

$$A_9 = 2\pi^2 L^6 [r^3 (T_1 T_2 T_3)^{-1/2}]_{r=\mu^{1/2}} = 2\pi^2 L^6 \mu^{3/2} \cosh \delta_1 \cosh \delta_2 \cosh \delta_3. \quad (37)$$

In the extreme limit it becomes

$$(A_9)_{BPS} = 2\pi^2 L^6 \sqrt{Q_1 Q_2 Q_3}. \quad (38)$$

The five-dimensional Einstein-frame metric obtained by reduction along y_1, \dots, y_6 is

$$ds_5^2 = -\lambda^2(r) f(r) dt^2 + \lambda^{-1}(r) [f^{-1}(r) dr^2 + r^2 d\Omega_3^2], \quad (39)$$

where

$$\lambda(r) = (T_1 T_2 T_3)^{1/3} = \frac{r^2}{[(r^2 + \mathcal{Q}_1)(r^2 + \mathcal{Q}_2)(r^2 + \mathcal{Q}_3)]^{1/3}}. \quad (40)$$

This is the metric of non-extreme five-dimensional black holes found in [16,28] (where one of the electric charges was replaced by a magnetic one). In the BPS limit $\mathcal{Q}_i \rightarrow Q_i$, $f \rightarrow 1$ and we get a solution which is U -dual to the solution of [4].

B. Intersection of Two-Brane and Five-Brane with a Boost

The non-extreme generalisation of the supersymmetric configuration of a two-brane intersecting five-brane with a “boost” along the common string [35] has the form

$$ds_{11}^2 = T^{-1/3} F^{-2/3} \left[T F (-K^{-1} f dt^2 + K \widehat{dy}_1^2) + T dy_2^2 + F (dy_3^2 + dy_4^2 + dy_5^2 + dy_6^2) \right] \quad (41)$$

$$+ f^{-1}dr^2 + r^2 d\Omega_3^2 \Big] ,$$

$$\mathcal{F}_4 = -3dt \wedge dT' \wedge dy_1 \wedge dy_2 + 3 * dF_1'^{-1} \wedge dy_2 , \quad (42)$$

where⁹ $\widehat{dy}_1 = dy_1 + (K'^{-1} - 1)dt$. The relevant functions of the radial coordinate r of the $(1+4)$ -dimensional space-time are

$$\begin{aligned} K &= 1 + \frac{\tilde{Q}}{r^2} , \quad K'^{-1} = 1 - \frac{\tilde{Q}}{r^2} K^{-1} , \quad \tilde{Q} = \mu \sinh^2 \beta , \quad \tilde{Q} = \mu \sinh \beta \cosh \beta , \\ T^{-1} &= 1 + \frac{\mathcal{Q}}{r^2} , \quad T' = 1 - \frac{\mathcal{Q}}{r^2} T , \quad \mathcal{Q} = \mu \sinh^2 \delta , \quad \mathcal{Q} = \mu \sinh \delta \cosh \delta , \\ F^{-1} &= 1 + \frac{\mathcal{P}}{r^2} , \quad F'^{-1} = 1 + \frac{P}{r^2} , \quad \mathcal{P} = \mu \sinh^2 \gamma , \quad P = \mu \sinh \gamma \cosh \gamma , \end{aligned} \quad (43)$$

and f is the same as in (36). The three charges $\tilde{Q}, \mathcal{Q}, P$ are held fixed in the extreme limit $\mu \rightarrow 0, \beta \rightarrow \infty, \delta \rightarrow \infty, \gamma \rightarrow \infty$.

We find again

$$A_9 = 2\pi^2 L^6 [r^3 (K^{-1} T F)^{-1/2}]_{r=\mu^{1/2}} = 2\pi^2 L^6 \mu^{3/2} \cosh \beta \cosh \delta \cosh \gamma , \quad (44)$$

$$(A_9)_{BPS} = 2\pi^2 L^6 \sqrt{\tilde{Q} \mathcal{Q} P} . \quad (45)$$

The corresponding five-dimensional Einstein-frame metric is (39) with

$$\lambda(r) = (TFK^{-1})^{1/3} = \frac{r^2}{\left[(r^2 + \tilde{Q})(r^2 + \mathcal{Q})(r^2 + \mathcal{P}) \right]^{1/3}} , \quad (46)$$

i.e. is precisely the space-time metric found in [16,28]. In the extreme limit $\tilde{Q} \rightarrow \tilde{Q}, \mathcal{Q} \rightarrow \mathcal{Q}, \mathcal{P} \rightarrow P, f \rightarrow 1$ we get back to the solution of [4].

V. $6 \leq D \leq 9$ NON-EXTREME BLACK HOLES

The generating solution for black holes in dimensions $D \geq 6$ can be parameterised by two charges. The boosted non-extreme two-brane naturally reduces to $D = 9$ black hole. The $D = 7$ non-extreme black hole can be described as a dimensional reduction of a configuration of two two-branes intersecting at a point. The boosted non-extreme five-brane represents the two-charge black hole in $D = 6$. Black holes in $D = 10$ do not have a natural M -brane description.

⁹Note that $dt \wedge dy_1$ remains invariant under the boost, i.e. \mathcal{F}_4 does not change.

A. Two-Brane with a Boost

It is possible to describe all two-charge $6 \leq D \leq 9$ non-extreme black holes as dimensional reductions of a non-extreme generalisation of boosted two-brane solution which has non-maximal rotational isometry, i.e. $O(D-1) \times [O(2)]^{9-D}$ symmetry, instead of $O(8)$. Adding a boost along one of the two directions of the two-brane we find from (9),(10)¹⁰

$$ds_{11}^2 = T^{-1/3} \left[T(-K^{-1}f dt^2 + K \widehat{dy}_1^2 + dy_2^2) + dy_3^2 + \dots + dy_{11-D}^2 \right. \\ \left. + f^{-1} dr^2 + r^2 d\Omega_{D-2}^2 \right], \quad (47)$$

$$\mathcal{F}_4 = -3dt \wedge dT' \wedge dy_1 \wedge dy_2, \quad (48)$$

where $\widehat{dy}_1 = dy_1 + (K'^{-1} - 1)dt$, and

$$f = 1 - \frac{\mu}{r^{D-3}}, \\ K = 1 + \frac{\tilde{Q}}{r^{D-3}}, \quad K'^{-1} = 1 - \frac{\tilde{Q}}{r^{D-3}} K^{-1}, \quad \tilde{Q} = \mu \sinh^2 \beta, \quad \tilde{Q} = \mu \sinh \beta \cosh \beta, \\ T^{-1} = 1 + \frac{Q}{r^{D-3}}, \quad T' = 1 - \frac{Q}{r^{D-3}} T, \quad Q = \mu \sinh^2 \delta, \quad Q = \mu \sinh \delta \cosh \delta. \quad (49)$$

Q and \tilde{Q} are the two electric charges which are held fixed in the extreme limit.

The area of the horizon at $r = \mu^{1/(D-3)}$ is

$$A_9 = \omega_{D-2} L^{11-D} [r^{D-2} (K^{-1} T)^{-1/2}]_{r=\mu^{1/(D-3)}} = \omega_{D-2} L^{11-D} \mu^{\frac{D-2}{D-3}} \cosh \beta \cosh \delta, \quad (50)$$

where all internal coordinates y_1, \dots, y_{11-D} are assumed to have period L and $\omega_{D-2} = 2\pi^{\frac{D-1}{2}}/\Gamma(\frac{D-1}{2})$. The area (50) vanishes in the BPS-saturated limit, in agreement with the fact that there are no BPS-saturated black holes with regular horizons of finite area in $D \geq 6$ [27,6].

The corresponding D -dimensional Einstein-frame metric is

$$ds_D^2 = -\lambda^{D-3}(r) f(r) dt^2 + \lambda^{-1}(r) [f^{-1}(r) dr^2 + r^2 d\Omega_{D-2}^2], \quad (51)$$

$$\lambda(r) = (K^{-1} T)^{\frac{1}{D-2}} = \frac{r^{\frac{2(D-3)}{D-2}}}{[(r^{D-3} + \tilde{Q})(r^{D-3} + Q)]^{\frac{1}{D-2}}}. \quad (52)$$

It coincides with the metric of non-extreme black hole solutions in $D \geq 6$ [25,26,6].

Note also that for $D = 10$ and $Q = 0$ the metric (51) describes also the electric R-R black hole (0-brane) in ten dimensions [45] which can be obtained by dimensional reduction of "boosted" Schwarzschild solution in $D = 11$.

¹⁰To construct such a non-extreme solution one should start with the extreme two-brane background and assume that the harmonic function does not depend on $D-9$ out of 9 transverse space coordinates, here denoted by y_3, \dots, y_{11-D} , or, equivalently, to consider a periodic array of two-branes in these directions.

B. Five-Brane with a Boost

The $D = 6$ black hole with one electric and one magnetic charge has a natural interpretation as a dimensional reduction of a boosted five-brane. Boosting the metric (12) along y_1 we find

$$ds_{11}^2 = F^{-2/3} \left[F(-K^{-1}f dt^2 + K \widehat{dy}_1^2 + dy_2^2 + \dots + dy_5^2) + f^{-1}dr^2 + r^2 d\Omega_4^2 \right], \quad (53)$$

where f and F are the same as in (14), $\widehat{dy}_1 = dy_1 + (K'^{-1} - 1)dt$ and

$$K = 1 + \frac{\tilde{Q}}{r^3}, \quad K'^{-1} = 1 - \frac{\tilde{Q}}{r^3} K^{-1}, \quad \tilde{Q} = \mu \sinh^2 \beta, \quad \tilde{Q} = \mu \sinh \beta \cosh \beta. \quad (54)$$

The black hole background resulting upon dimensional reduction along internal five-brane directions y_1, \dots, y_5 is parameterised by one electric and one magnetic charge.

C. Intersection of two Two-Branes

The $D = 7$ non-extreme black hole admits also a description in terms of non-extreme version of $2 \perp 2$ configuration.¹¹ Dimensional reduction of the background (15), (16) along y_1, \dots, y_4 leads to the $D = 7$ black hole background, with the role of $\mathcal{Q}, \tilde{\mathcal{Q}}, Q, \tilde{Q}$ played by $\mathcal{Q}_1, \mathcal{Q}_2, Q_1, Q_2$.

It is also possible to give an alternative description of $D = 6$ black hole (now with two electric charges) by using $O(5)$ -symmetric version of (15). i.e. the non-extreme version of $2 \perp 2$ solution with one of the transverse space coordinates (y_1) treated as an isometric internal space one:

$$ds_{11}^2 = (T_1 T_2)^{-1/3} \left[-T_1 T_2 f dt^2 + dy_1^2 + T_1(dy_2^2 + dy_3^2) + T_2(dy_4^2 + dy_5^2) + f^{-1}dr^2 + r^2 d\Omega_4^2 \right]. \quad (55)$$

The corresponding nine-area and the $D = 6, 7$ Einstein-frame metrics reduce to the expressions in (50) and (51), with the role of T, K^{-1} now played by T_1, T_2 .

It is also of interest to compare the metrics of the eleven-dimensional solutions which reduce to black holes in $D = 4, 5, 6$ with respective $n = 4, 3, 2$ charges in the case when all charges (boost parameters) are equal, i.e. $T_i = F_i = K^{-1} = H^{-1}$. For $n = 4$ and $n = 3$, i.e. the respective cases of $D = 4$ and $D = 5$ black holes, whose extreme limits have *regular* horizons, we get

¹¹A natural description of two-charge $D = 7$ black hole in type IIB theory is given in terms of compactified boosted three-brane [46].

$$ds_{11}^2 = H^{n-2} \left[-H^{-n} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{6-n}^2 \right] + \widehat{dy}_1^2 + \dots + dy_{n+3}^2, \quad (56)$$

where $\widehat{dy}_1 = dy_1$ for the unboosted configurations (18), (34), and $\widehat{dy}_1 = dy_1 + (H'^{-1} - 1)dt$ for the boosted ones (26), (41). At the same time, in the case of $n = 2$, i.e. black holes whose extreme limits have singular horizons, e.g., for $D = 6$ black hole, we get the metric

$$ds_{11}^2 = H^{2/3} \left[H^{-1} (-f dt^2 + dy_2^2 + \dots + dy_5^2) + \widehat{dy}_1^2 + f^{-1} dr^2 + r^2 d\Omega_4^2 \right], \quad (57)$$

where $\widehat{dy}_1 = dy_1$ for the intersecting two-brane representation (55) and $\widehat{dy}_1 = dy_1 + (H'^{-1} - 1)dt$ for the boosted five-brane one (53). While the radii of the internal coordinates are constant in the case (56), i.e. $D = 4, 5$ black holes with equal $n = 3, 4$ charges, this is no longer so for black holes in $D > 5$ with equal $n = 2$ charges.¹²

VI. UNIVERSAL EXPRESSIONS FOR MASS AND ENTROPY

It is possible to write down the formulas for the mass and the entropy which apply to all eleven-dimensional “anisotropic p-brane” solutions discussed in previous Sections. Similar expressions for the corresponding non-extreme black holes were given in [44,6]. Note also that analogous relations for isotropic black brane solutions appeared in [42,27].

Let $p = 11 - D$ be the common internal dimension of intersecting M-branes, i.e. the dimension of an anisotropic p-brane. Then D is the dimension of the black hole obtained by dimensional reduction. For $D = 4$, $D = 5$ and $6 \leq D \leq 9$ the black hole has the metric of the form (51) with $\lambda(r) = (H_1 \dots H_n)^{-1/(D-2)}$, $H_i = 1 + \mathcal{Q}_i/r^{D-3}$, $i = 1, \dots, n$, with $n \leq 4$, $n \leq 3$ and $n \leq 2$, respectively (cf. (25), (33), (40), (46), (52)). Here we shall use the same notation \mathcal{Q}_i for all n charges, some of which may be electric, and some magnetic. Let $G_{11} = 8\pi\kappa^2$ be the Newton’s constant in eleven dimensions (the Newton’s constant in D dimensions is then $G_D = G_{11}/L^p$). Then for a $O(D-1)$ -symmetric solution the normalised charges per unit volume are¹³

$$q_i = a\mathcal{Q}_i, \quad a \equiv \frac{\omega_{D-2}}{\sqrt{2\kappa}}(D-3), \quad \mathcal{Q}_i = \mu \sinh\beta_i \cosh\beta_i, \quad \mathcal{Q}_i = \mu \sinh^2\beta_i. \quad (58)$$

As follows from (52), the ADM mass is

$$M_{ADM} = \frac{\omega_{D-2}}{2\kappa^2} L^p [(D-2)\mu + (D-3) \sum_{i=1}^n \mathcal{Q}_i]. \quad (59)$$

¹²Note that “bound-state” metrics (56), (57) are different from the isotropic black p-brane ones discussed in [42,27].

¹³The relation of our notation to that of [27] is the following: $D-3 \rightarrow d$, $\mathcal{Q} \rightarrow r_0^d$, $\mathcal{Q} \rightarrow r_-^d$, $\mu \rightarrow \mu^d$. The charges q_i correspond to constituent objects of an N -charge bound state (hence there is no \sqrt{n} factor in the expression for the charges).

It can be expressed in terms of the non-extremality parameter μ and charges Q_i (which are fixed in the extreme limit $\mu \rightarrow 0$) as follows (note that $Q_i = -\frac{\mu}{2} + \sqrt{Q_i^2 + (\frac{\mu}{2})^2}$)

$$M_{ADM} = b \left[\sum_{i=1}^n \sqrt{Q_i^2 + (\frac{\mu}{2})^2} + \lambda\mu \right], \quad b \equiv \frac{\omega_{D-2}}{2\kappa^2} (D-3)L^p, \quad (60)$$

where the parameter λ (not to be confused with the function $\lambda(r)$ in D -dimensional metric (51)) is the same as in [27]

$$\lambda \equiv \frac{D-2}{D-3} - \frac{n}{2}. \quad (61)$$

Explicitly, $\lambda_{n=1} = \frac{D-1}{2(D-3)}$, $\lambda_{n=2} = \frac{1}{D-3}$, $\lambda_{n=3} = \frac{5-D}{2(D-3)}$, $\lambda_{n=4} = \frac{4-D}{2(D-3)}$, i.e. $\lambda \geq 0$ and *vanishes* only for $D=4$, $n=4$ and $D=5$, $n=3$, i.e. the cases with regular horizons in the extreme (BPS-saturated) limit.¹⁴

The Bekenstein-Hawking entropy S_{BH} , which follows from the obvious generalisation of the expressions for the area (50), (22),(31),(37) and (44) (with $K^{-1}T \rightarrow (H_1 \dots H_n)^{-1}$) is

$$S_{BH} = \frac{2\pi A_9}{\kappa^2} = c\mu^{\frac{D-2}{D-3}} \prod_{i=1}^n \cosh\delta_i, \quad c \equiv \frac{2\pi\omega_{D-2}}{\kappa^2} L^p, \quad (62)$$

or, in terms of the Hawking temperature T_H ,

$$S_{BH} = b \frac{\mu}{T_H}, \quad T_H = \frac{1}{4\pi} (D-3)\mu^{-\frac{1}{D-3}} \prod_{i=1}^n (\cosh\delta_i)^{-1}. \quad (63)$$

These general formulas apply also to the case of $D=10$ black hole [45] where $D=10$ and $n=1$.

Expressed in terms of μ and Q_i , the entropy becomes

$$S_{BH} = c\mu^\lambda \prod_{i=1}^n \left[\sqrt{Q_i^2 + (\frac{\mu}{2})^2} + \frac{\mu}{2} \right]^{1/2}. \quad (64)$$

It has non-zero extreme limit ($\mu \rightarrow 0$) only when $\lambda=0$. In this case M_{ADM} and S_{BH} take simple forms

$$M_{ADM} = b \sum_{i=1}^n \sqrt{Q_i^2 + (\frac{\mu}{2})^2}, \quad S_{BH} = c \prod_{i=1}^n \left[\sqrt{Q_i^2 + (\frac{\mu}{2})^2} + \frac{\mu}{2} \right]^{1/2}. \quad (65)$$

M_{ADM} resembles the energy of a system of relativistic particles with masses Q_i (masses of individual constituents in extreme limit), all having the same momentum proportional to μ . This suggests a “bound-state” interpretation of this non-extreme system (cf. [47]).

¹⁴The above expressions for the charges and the mass include all previously discussed non-extreme cases: single two-brane ($p=2$, $D=9$), single five-brane ($p=5$, $D=6$), and equal-charge $p=4$ ($D=7$) and $p=6$ ($D=5$) anisotropic branes [43]. Note that our expression for the mass in $p=4$ case disagrees with that in [43].

For all fixed values of λ and Q_i the mass (60) and the entropy (64) satisfy the following relation

$$\frac{\partial \ln S_{BH}}{\partial \ln \mu} = b^{-1} \frac{\partial M_{ADM}}{\partial \mu} , \quad i.e. \quad T_H \frac{\partial S_{BH}}{\partial \mu} = \frac{\partial M_{ADM}}{\partial \mu} . \quad (66)$$

This thermodynamic relation is valid both in the Schwarzschild ($Q_i = 0$) case as well as in the extreme limit ($\mu = 0$). This explains why the proportionality between the entropy and the area of the horizon can be assumed to be true also in the extreme limit (cf. [15]): this proportionality certainly holds in non-extreme (or near-extreme) case (see, e.g., [48]) and thus should be meaningful also in the limit $\mu \rightarrow 0$.

Other intersecting M-brane configurations (with $\lambda \neq 0$) have zero entropy in the extreme limit ($\mu \rightarrow 0$). Representative examples of such configurations are unboosted $p = 7$ configurations $5 \perp 5 \perp 5$, $5 \perp 5 \perp 2$, $5 \perp 2 \perp 2$, corresponding to $D = 4$ black holes with $n = 3$ charges, unboosted $p = 6$ configuration $2 \perp 5$, corresponding to $D = 5$ black hole with $n = 2$ charges, and $p = 4$ configuration $2 \perp 2$, corresponding to $D = 7$ black hole with $n = 2$ charges. In this case it is of interest to study the near-extreme limit, where

$$M_{ADM} = M_0 + \Delta M + O(\mu^2) , \quad M_0 = b \sum_{i=1}^n Q_i , \quad \Delta M = b \lambda \mu , \quad (67)$$

$$S_{BH} = c_1 \prod_{i=1}^n Q_i^{1/2} E^\lambda , \quad E \equiv \Delta M , \quad c_1 = c(b\lambda)^{-\lambda} . \quad (68)$$

Using the thermodynamic relation $dE = T dS$ it follows from (68) that

$$S_{BH} = c_2 \prod_{i=1}^n (Q_i)^{\frac{1}{2(1-\lambda)}} T^{\frac{\lambda}{1-\lambda}} , \quad c_2 = (c_1 \lambda^\lambda)^{\frac{1}{1-\lambda}} = (c b^{-\lambda})^{\frac{1}{1-\lambda}} , \quad (69)$$

where

$$T = (c_1 \lambda)^{-1} \prod_{i=1}^n Q_i^{-1/2} E^{1-\lambda} \quad (70)$$

is the near-extreme limit of the Hawking temperature T_H in (63).

Generalising the discussion in [27], we may enquire when this entropy has a massless ideal gas entropy form. The power $\nu = \lambda/(1-\lambda)$ of the temperature in (69) is equal to 2 and 5 for unboosted two-brane and five-brane, respectively [27].¹⁵ The only other cases when ν is integer are configurations with $D = 5$, $n = 2$, i.e. the anisotropic six-brane corresponding to unboosted $2 \perp 5$ intersection, and $D = 4$, $n = 3$, i.e. the anisotropic seven-branes corresponding to unboosted $5 \perp 5 \perp 5$, $2 \perp 5 \perp 5$ and $2 \perp 2 \perp 5$ intersections. In these cases

¹⁵In particular, for the five-brane ($D = 6, n = 1, \lambda = \frac{5}{6}, \nu = 5$) eq. (69) gives the expression found in [27] $S_{BH} = 2^7 3^{-6} \pi^3 n^3 L^5 T^5$ after one uses the charge quantisation condition $q = \frac{3\omega_4}{\sqrt{2}\kappa} Q = (\frac{\sqrt{2}\pi}{\kappa})^{1/3} n$.

$\lambda = 1/2$ and $\nu = 1$, so that, not unexpectedly [33], here S_{BH} has a “string”-like form, i.e. the form of the entropy of a gas of massless particles in (1+1)-dimensions. For other anisotropic eleven-dimensional p-branes, e.g., those reducing to isotropic dilatonic p-branes in lower dimensions [42,27], S_{BH} does not have an ideal gas scaling.

To conclude, we have constructed non-extreme versions of intersecting M-brane solutions which correspond to one parameter “deformations” of the supersymmetric intersecting M-brane solutions, and maintain the simple “product” structure. This product structure implies that the non-extreme static black holes obtained upon dimensional reduction have a form which provides a straightforward interpolation between the Schwarzschild and BPS-saturated backgrounds.

The M-brane interpretation of non-extreme black hole solutions may provide an insight into the problem of statistical understanding of their properties. In particular, it would be of interest to study in more detail the statistical origin of the BH entropy of near-extreme anisotropic p-branes and interpret them in terms of massless modes living on near-extreme intersections, along the lines of [27,33].

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